BAULKHAM HILLS HIGH SCHOOL

2016 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11–16, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks – 100
Exam consists of 13 pages.

This paper consists of TWO sections.

Section 1 – Page 2-5 (10 marks)

- Attempt Question 1-10
- Allow about **15** minutes for this section

Section II – Pages 6-13 (90 marks)

- Attempt questions 11-16
- Allow about **2 hours and 45** minutes for this section

The reference sheet is on page 14.

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section.
Use the multiple choice answer sheet for questions 1-10

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1. What is 0.0050279 written in scientific notation correct to 3 significant figures?

(A) 5.02×10^{-2}

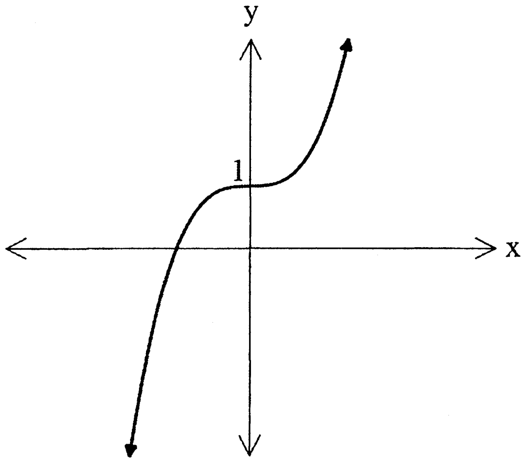
(B) 5.03×10^{-2}

(C) 5.02×10^{-3}

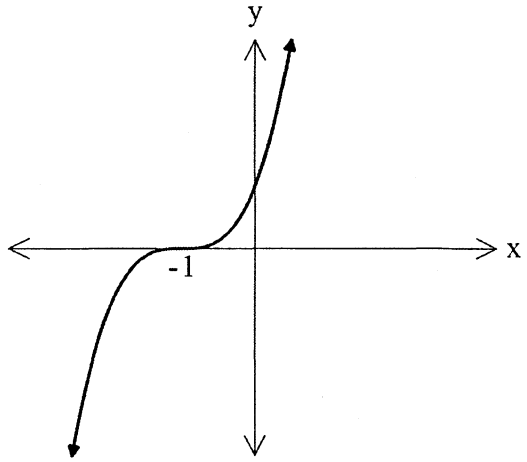
(D) 5.03×10^{-3}

2. The graph of $y = 1 - x^3$ could be:

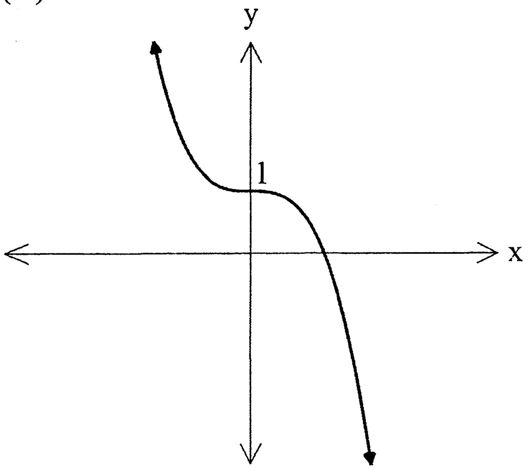
(A)



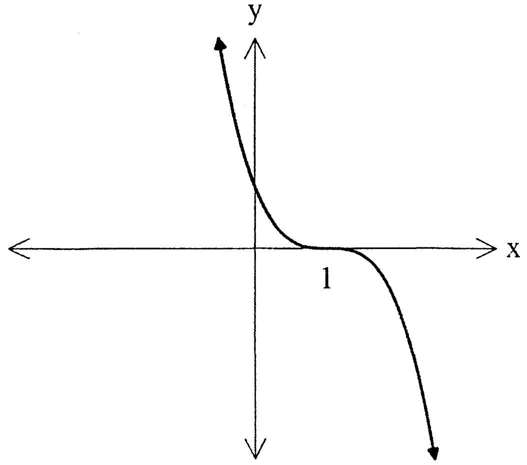
(B)



(C)



(D)



3. What is the perpendicular distance of the point $(2,-1)$ from the line $y = 3x + 1$?

(A) $\frac{6}{\sqrt{10}}$

(B) $\frac{6}{\sqrt{5}}$

(C) $\frac{8}{\sqrt{10}}$

(D) $\frac{8}{\sqrt{5}}$

4. $(2\sqrt{5} - \sqrt{3})^2 =$

(A) 17

(B) 23

(C) $17 - 4\sqrt{15}$

(D) $23 - 4\sqrt{15}$

5. A parabola has a focus of $(-3,0)$ and a directrix of $x = 1$. What is the equation of the parabola?

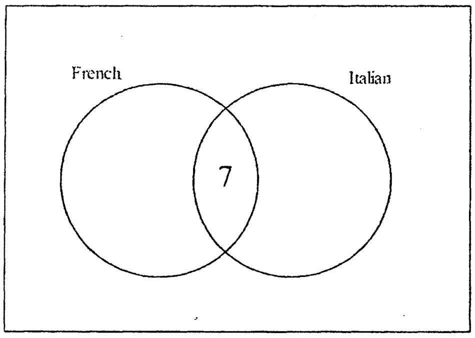
(A) $y^2 = 16(x + 3)$

(B) $y^2 = -16(x + 3)$

(C) $y^2 = 8(x + 1)$

(D) $y^2 = -8(x + 1)$

6. The Venn Diagram represents a group of 30 students all of whom study either Italian, French or both languages. 7 students study both and 19 students study French.



By completing the Venn Diagram or otherwise, find the probability that if 2 people are selected at random that they only study Italian.

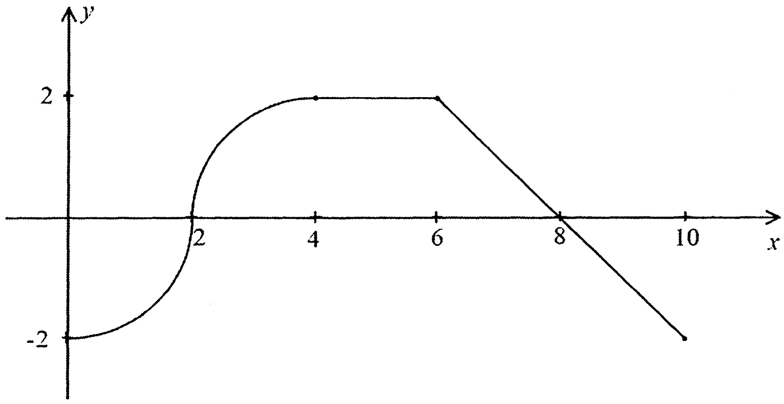
(A) $\frac{11}{87}$

(B) $\frac{2}{145}$

(C) $\frac{22}{145}$

(D) $\frac{57}{145}$

7. The graph of $y = f(x)$ is drawn below, showing two quadrants and two line segments.



Which of the following is true?

(A) $\int_0^{10} f(x)\ dx = 2\pi + 8$ and the curve is differentiable at $x = 2$

(B) $\int_0^{10} f(x)\ dx = 4$ and the curve is differentiable at $x = 2$

(C) $\int_0^{10} f(x)\ dx = 2\pi + 8$ and the curve is not differentiable at $x = 2$

(D) $\int_0^{10} f(x)\ dx = 4$ and the curve is not differentiable at $x = 2$

8. Which inequality defines the domain for $= \frac{1}{\sqrt{x^2-9}}$?

(A) $x < -3$ or $x > 3$

(B) $x \leq -3$ or $x \geq 3$

(C) $-3 < x < 3$

(D) $-3 \leq x \leq 3$

9. What is the value of $\int_1^4 \frac{1}{3x} dx$?

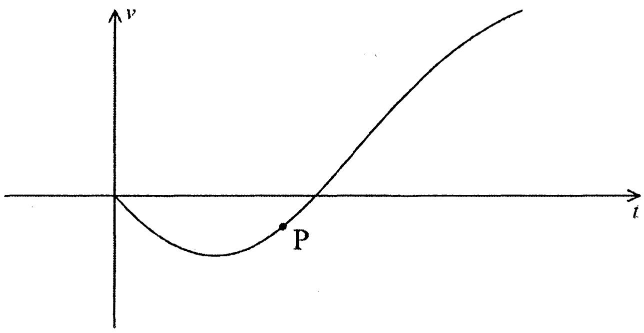
(A) $\frac{1}{3}\ln 3$

(B) $\frac{1}{3}\ln 4$

(C) $\ln 9$

(D) $\ln 12$

10. The graph shows the velocity of a particle moving along a straight line as a function of time.



Which statement describes the motion of the particle at the point P.

(A) The particle is moving left at increasing speed.

(B) The particle is moving left at decreasing speed.

(C) The particle is moving right at decreasing speed.

(D) The particle is moving right at increasing speed.

END OF SECTION I

Section II

90 marks

Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section
Answer each question in the appropriate page in the writing booklet.
In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 **(15 marks)** - Start on the appropriate page in your answer booklet **Marks**

a) Simplify

(i) $\frac{1-x}{3}+\frac{x}{4}$ **1**

(ii) $\frac{8^{x-2}}{2^{3-x}}$ **2**

b) Factorise fully $16-36x^2$ **2**

c) Two 6 sided dice are rolled. What is the probability that a 5 or a 4 is on the upper most face? **2**

d) Differentiate

(i) $2\sin 3x$ **2**

(ii) $(2x+1)^8$ **2**

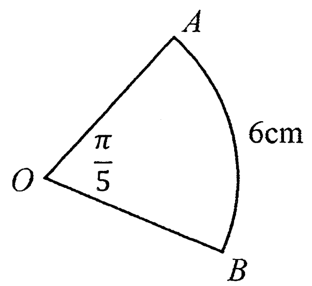
e) Evaluate $\int_0^2 e^{2x}\,dx$ **2**

f) Find $\int 2cos\pi x\,dx$ **2**

End of Question 11

Question 12 **(15 marks)** - Start on the appropriate page in your answer booklet. **Marks**

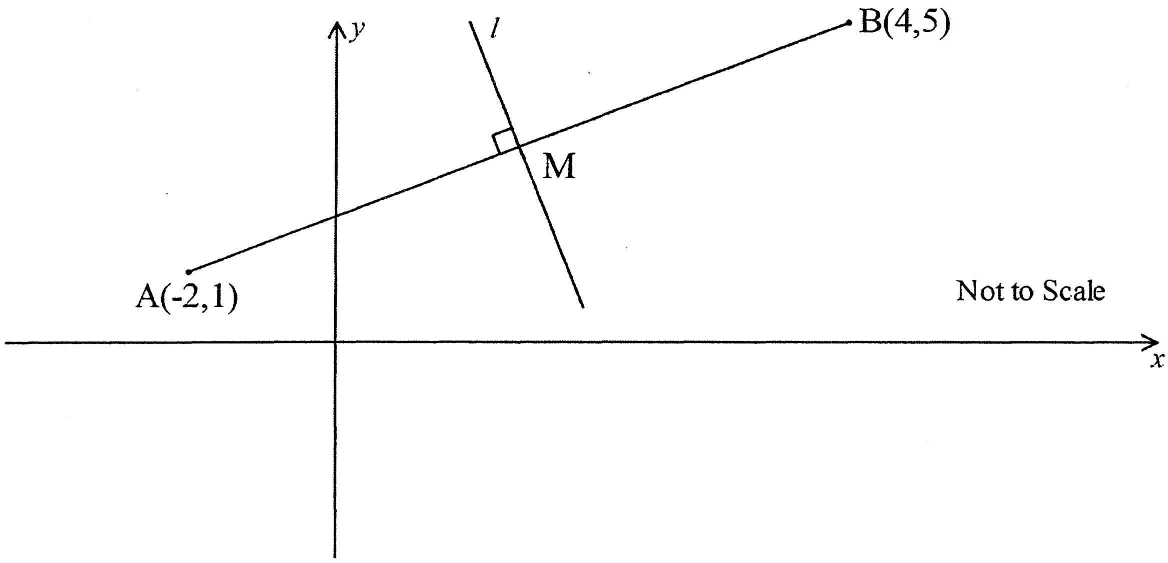
a) The sector shown has arc length of 6 cm and $\angle AOB$ is $\frac{\pi}{5}$ radians.



(i) Find the radius of the circle in terms of π. **1**

(ii) Hence, find the exact area of the sector. **1**

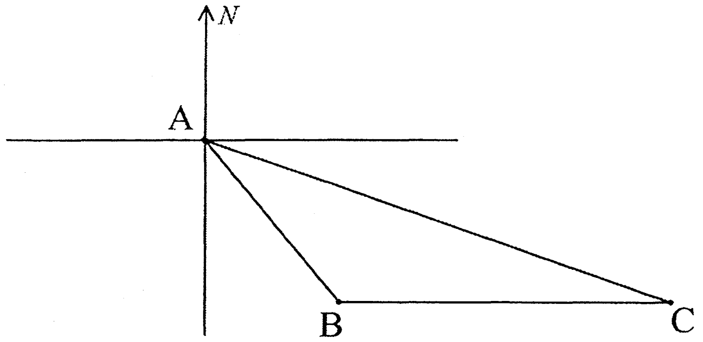
b) The interval AB is drawn where $A = (-2,1)$ and $B = (4,5)$ and the line l is perpendicular to AB passing through M.



(i) Show that M the midpoint of AB has coordinates $(1,3)$. **1**

(ii) Show that the equation of the line l, perpendicular to AB passing through M, is given by $3x + 2y - 9 = 0$. **3**

c) The diagram below, represents the journey taken by a ship which leaves point A and travels 200km on a bearing of 112° to B. It then turns and travels 150 km due east to C.



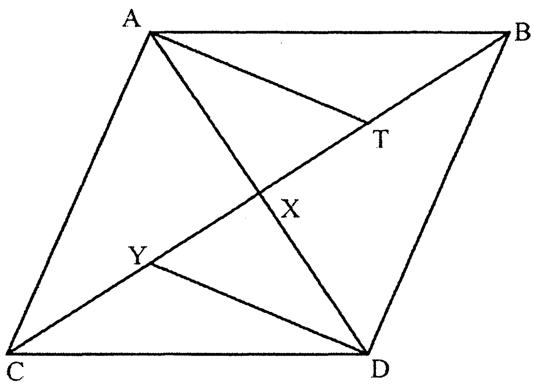
(i) Draw a neat sketch of the diagram above in your answer booklet.

(ii) Show $\angle ABC = 158°$ **1**

(iii) Find the distance AC. **2**

d) The roots of the quadratic equation $x^2 - kx - 6 = 0$ are α and β. Find k if $\alpha^2\beta + \alpha\beta^2 = 4$. **2**

e) ABDC is a rhombus whose diagonals intersect at X.
Y and T lie on BC such that AT || DY.



(i) Draw a neat sketch of the diagram above in your answer booklet.

(ii) Prove $\Delta AXT \equiv \Delta XYD$. **3**

(iii) Prove $ATDY$ is a parallelogram. **1**

End of Question 12

Question 13 **(15 marks)** - Start on the appropriate page in your answer booklet **Marks**

a) Consider the curve $y = x^3 - 6x^2 - 15x + 2$

(i) Find the stationary points and determine their nature. **3**

(ii) Find the point of inflexion. **2**

(iii) Sketch the curve labelling the stationary points, the point of inflexion and y intercept. **2**

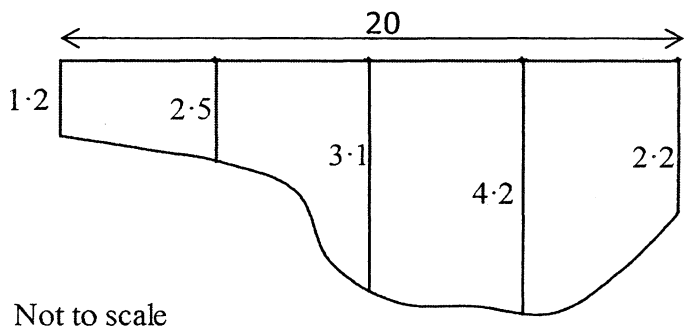
b) 150 koalas were introduced to an isolated island.
The rate at which the population (P) of the koalas increase is proportional to the population according to the differential equation,
$\frac{dP}{dt} = 0.02P$ where t is measured in years.

(i) Show $P = Ae^{0.02t}$ is a solution to the differential equation above. **1**

(ii) How many koalas are on the island after 10 years? **1**

(ii) How many years will it take for the population of the koalas to reach 200? **2**

c) At a certain location, a river is 20 metres wide. At this location the depth of the river in metres has been measured at 5 metre intervals.
The cross section of the river is shown below.



(i) Use Simpson's Rule with the 5 depth measurements to calculate the approximate area of the cross section. **3**

(ii) The river flows at 0.6m/sec. Calculate the approximate volume of water flowing through the cross section of the river in 10 seconds. **1**

End of Question 13

Question 15 **(15 marks)** - Start on the appropriate page in your answer booklet **Marks**

a) The acceleration of a particle moving along the x axis is given by $\ddot{x} = 4 \sin 2t$ where x is the displacement from the origin in metres and t is the time in seconds. Initially the particle is at the origin moving to the left at 1 metre/second

(i) Show the velocity is given by $\dot{x} = 1 - 2\cos 2t$. **1**

(ii) Find the time when the particle first comes to rest. **2**

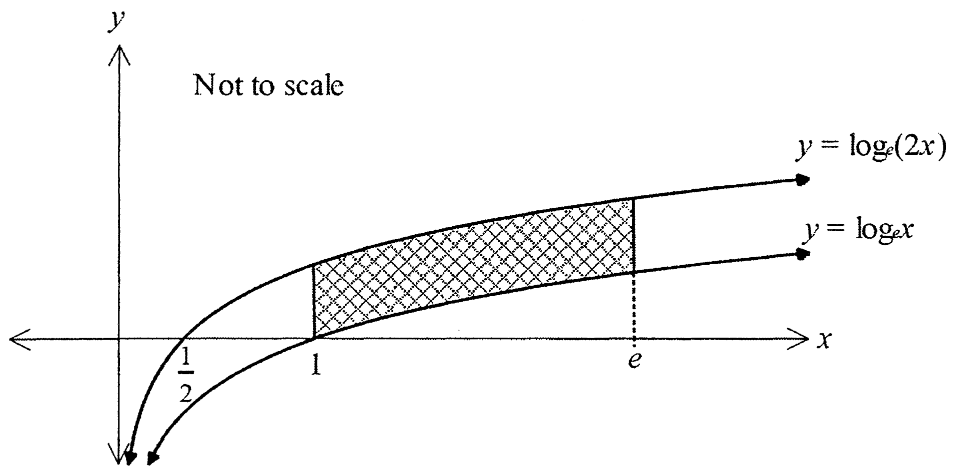
(iii) Show that $x = t - \sin 2t$. **1**

(iv) Find the distance travelled by the particle in the first $\frac{\pi}{2}$ seconds. **3**

b) (i) Show that $e^{1-\ln 2} = \frac{e}{2}$ **1**

(ii) Find the equation of the tangent to the curve $y = e^{1-4x}$ at $x = \frac{\ln 2}{4}$ **3**

c) The curves of $y = \ln x$ and $y = \ln 2x$ are drawn below. **4**

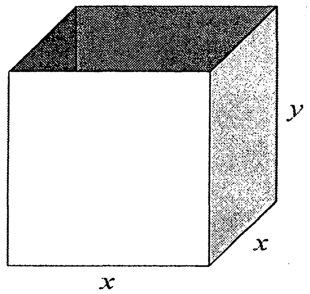


Find the shaded area between the curves $y = \ln 2x$ and $y = \ln x$ and the lines $x = 1$ and $x = e$.

End of Question 15

Question 16 **(15 marks)** - Start on the appropriate page in your answer booklet **Marks**

a) A rectangular box with a square base and **no** top is drawn below.



The volume of the box is 500cm^3

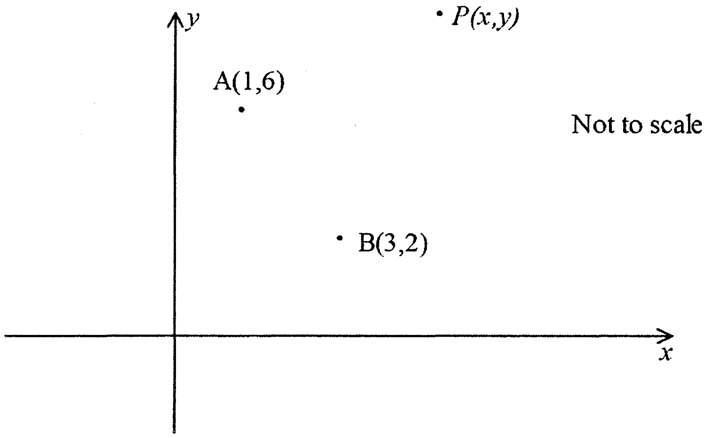
(i) Show that the surface area (A) of the box is given by $A = x^2 + \frac{2000}{x}$. **2**

(ii) Find the least area of sheet metal required to make the box. **3**

b) (i) The sequence $\sin\theta, 2\cos\theta, 2\sin\theta \ldots$ form the first three terms of an arithmetic progression. **3**
Find the value of θ to the nearest minute.

(ii) Find the next term in the sequence in terms of $\sin\theta$. **2**

c)



(i) Find the gradient of PA in terms of x and y. **1**

(ii) The point P moves such that the gradient of PB is twice the gradient of PA. Find the values of a, b and c when the locus is expressed in the form **3**

$$y = a + \frac{b}{x-c}$$

(iii) Sketch the locus of P. **1**

End of Exam

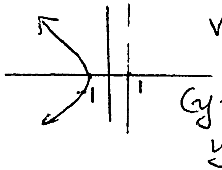
Trial Advanced 2016 Solutions:-

1. D 2. C 3. C 4. D 5. D 6. A
7. D 8. A 9. B 10. B

3/ $3x-y+1=0$ $(2,-1)$

$$d = \frac{|3(2)+(-1)(-1)+1|}{\sqrt{3^2+(-1)^2}} = \frac{8}{\sqrt{10}} \quad (C)$$

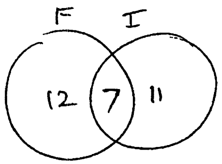
4/ $(2\sqrt{5}-\sqrt{3})^2 = 20 - 4\sqrt{15}+3$
$= 23 - 4\sqrt{15}$ (D)

5/ 

$V=(-1,0)$ $a=2$

$(y-0)^2 = -4(2)(x+1)$

$y^2 = -8(x+1)$ (C)

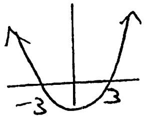
6/ 

$P(I,I) = \frac{11}{30} \times \frac{10}{29} = \frac{11}{87}$ (A)

7. $\int_0^{10} f(x) = -\frac{1}{4}\pi(2)^2 + \frac{1}{4}\pi(2)^2$
$+ \frac{1}{2}\times 2\times 2 + 2\times 2 - \frac{1}{2}\times 2\times 2$
$= 4$

Tangent is vertical ∴ not diff'ble
∴ (D)

8. $x^2-9>0$ $x<-3$ or $x>3$
(A)



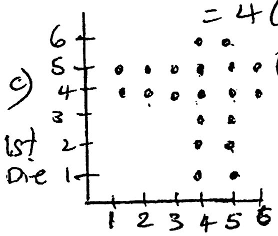
9. $\int_1^4 \frac{1}{3x}\,dx = \frac{1}{3}\int_1^4 \frac{1}{x}$
$= \frac{1}{3}(\ln x)_1^4$
$= \frac{1}{3}(\ln 4 - \ln 1)$
$= \frac{1}{3}\ln 4$ ...

10. at P. V is below x axis
∴ $V<0$ ∴ moving left.
Tangent at P is positive.
∴ $\ddot{x}>0$
∴ moving left & slowing down (B)

11a) (i) $\frac{1-x}{3} + \frac{x}{4} = \frac{4-4x+3x}{12}$
$= \frac{4-x}{12}$ ①

(ii) $\frac{8^{x-2}}{2^{3-x}} = \frac{2^{3x-6}}{2^{3-x}}$ → ①
$= 2^{3x-6-3+x}$
$= 2^{4x-9}$ ①

b) $16-36x^2 = 4(4-9x^2)$
$= 4(2-3x)(2+3x)$

c) 

P(4 or 5 on upper face)
$= \frac{20}{36} = \frac{5}{9}$ ①

(1 mark for $\left(\frac{24}{36} = \frac{2}{3}\right)$
$\left(\frac{22}{36} = \frac{11}{18}\right)$)

d) (i) $\frac{d}{dx}(2\sin 3x) = 6\cos 3x$ ① ① ①

(ii) $\frac{d}{dx}\left[(2x+1)^8\right] = 8(2x+1)^7 \times 2$
$= 16(2x+1)^7$ ①

e) $\int_0^2 e^{2x}dx = \frac{1}{2}\left[e^{2x}\right]_0^2$ ①
$= \frac{1}{2}(e^4-1)$ ①

f) $\int 2\cos \pi x\,dx = \frac{2}{\pi}\sin \pi x + C$

Question 12

a) (i) $l = r\theta$ ∴ $r = \frac{6}{\frac{\pi}{5}} = \frac{30}{\pi}$ ①
(9.549)✓

(ii) $A = \frac{1}{2}r^2\theta$
$= \frac{1}{2}\cdot\left(\frac{30}{\pi}\right)^2\cdot\frac{\pi}{5}$
$= \frac{900\pi}{10\pi^2} = \frac{90}{\pi}$ ① (28.648..)✓

b) (i) $M = \left(\frac{-2+4}{2}, \frac{5+1}{2}\right) = (1,3)$ ①

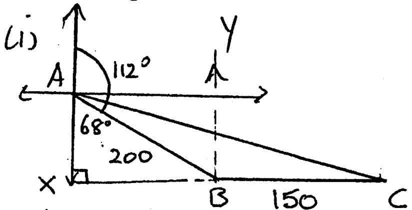
(ii) $m_{AB} = \frac{5-1}{4--2} = \frac{2}{3}$

⊥m $= -\frac{3}{2}$ - ①

$y-3 = -\frac{3}{2}(x-1)$ ①

$2y-6 = -3x+3$ - - ①

$3x+2y-9=0$

c) (i) 

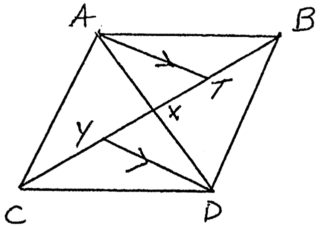
$\angle ABX = 22°$
$\angle ABC = 180-22$
$= 158°$
(No reasons necessary)

or $\angle ABY = 68°$ $\angle YBC = 90°$
∴ $\angle ABC = 158°$

(ii) $AC^2 = 200^2+150^2-2(200)(150)\cos 158°$ ①

$AC = 343.7$ km ①

d) $x^2-kx-6=0$ $\alpha+\beta=k$, $\alpha\beta=-6$ ①
$\alpha^2\beta+\alpha\beta^2=4$
$\alpha\beta(\alpha+\beta)=4$
... $k=-\frac{2}{3}$ ①

e) 

(i) $AX = XC$ (diagonals of a rhombus bisect each other) ①

$\angle TAX = \angle XDY$ (Alternate ∠'s AT∥DY)
$\angle XYD = \angle ATX$ (" " " ") ①

∴ $\triangle ATX \equiv \triangle XYC$ (AAS) ①

(ii) $XY = TX$ (Matching sides in ≡ Δ's)

∴ AD and TY bisect each other

∴ ATDY is a parallelogram (diagonals bisect each other)

(both statements must be made for ①)

Question 13.

a) $y = x^3 - 6x^2 - 15x + 2$

(i) St. pts $y' = 0$

$y' = 3x^2 - 12x - 15$

$3(x^2 - 4x - 5) = 0$ (1)

$3(x-5)(x+1) = 0$

$x = -1 \quad , \quad 5$

$y = 10 \quad y = -98$

$(-1, 10) \quad (5, -98)$ (1)

Test nature $y'' = 6x - 12$

when $x = -1$ $y'' = -18 < 0$ $\therefore$ max (1)

when $x = 5$ $y'' = 18 > 0$ $\therefore$ min. (1)

(ii) Point of inflexion when when $y'' = 0$ + concavity change.

$y'' = 6x - 12 = 0$

when $x = 2$ (1)

$y = -44$ $(2, -44)$

Test change in concavity

at $x = -1$ $y'' = -18 < 0$

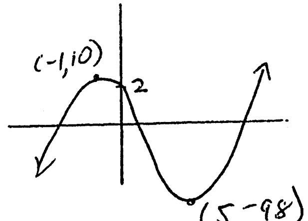
at $x = 5$ $y'' = 18 > 0$ (1)

change in concavity $\therefore$ point of inflexion.

b(i) $P = Ae^{0.02t}$ - - (1)

$\frac{dP}{dt} = 0.02(Ae^{0.02t})$

[illegible] $\frac{dP}{dt} = 0.02P$



must have shape & y intercept (1)

(1) points

(i) See bottom of first column.

b) (ii) $P = P_0 e^{0.02t}$

when $t = 0$ $P = 150$

$\therefore 150 = P_0 e^0 \rightarrow P_0 = 150$

$P = 150 e^{0.02t}$ (1)

when $t = 10$

$P = 150 e^{0.2}$ (1)

$P = 183$ (ie 183 koalas)

(iii) find t when $P = 200$

$200 = 150 e^{0.02t}$

$\frac{4}{3} = e^{0.02t}$

$\ln\left(\frac{4}{3}\right) = 0.02t$ ~~lne~~ (1)

$t = \frac{\ln\left(\frac{4}{3}\right)}{0.02}$

$t = 14.38\ldots$ or (1)

$\therefore$ In 15^{th} year.

c) (i) Area $= \frac{5}{3}\left[1.2 + 2.2 + 4(2.5 + 4.2) + 2(3.1)\right]$ (1) (1)

$= 60\frac{2}{3}\ \text{m}^2$ (1)

(ii) Volume $= 60\frac{2}{3} \times 6$

$= 364\ \text{m}^3$ (1)

Question 14

a) (i) $800 \times 1.04^{14} = 1385.34$

(1) (1)

(ii) Total Investments =

$800 \times 1.04^{14} + 800 \times 1.04^{13} + \cdots$

$800 \times 1.04^1 + 800$ (1)

$S_n = \frac{800(1.04^{15} - 1)}{1.04 - 1}$ (1) (1)

$= \$16018.87$

(iii) $8000\left(1 + \frac{r}{100}\right)^{14} = 16018.87$ (1)

$1 + \frac{r}{100} = \sqrt[14]{\frac{16018.87}{8000}}$

$1 + \frac{r}{100} = 1.0508\ldots$

$\therefore r = 5.08\%$ (1)

b) (i) $y = \frac{1 + \sqrt{x}}{2}$

$\sqrt{x} = 2y - 1$ (1)

$\therefore x = (2y-1)^2$

(ii) Volume $=$

$\pi \int_{\frac{1}{2}}^{3} x^2\, dy$

$= \pi \int_{\frac{1}{2}}^{3} (2y-1)^4\, dy$ (1)

$= \pi \left[\frac{(2y-1)^5}{10}\right]_{\frac{1}{2}}^{3}$ (1)

$= \frac{\pi}{10}\left[5^5 - 0\right]$

$= \frac{3125\pi}{10} = \frac{625\pi}{2}$

c)(i) $y = x^2 - 6x + 8$

roots $0 = x^2 - 6x + 8$

$0 = (x-2)(x-4)$ (1)

$\therefore x$ intercepts are 2 & 4.

Area $= \int_0^2 x^2 - 6x + 8 + \left|\int_2^4 x^2 - 6x + 8\right|$ (1)

$= \left(\frac{x^3}{3} - 3x^2 + 8x\right)_0^2 + \left|\left(\frac{x^3}{3} - 3x^2 + 8x\right)_2^4\right|$

$= \left[\left(\frac{8}{3} - 12 + 16\right) - (0)\right]$

$+ \left|\left(\frac{64}{3} - 48 + 32\right) - \left(\frac{8}{3} - 12 + 16\right)\right|$

$= \frac{20}{3} + \left|\frac{16}{3} - \frac{20}{3}\right|$

(1)

$= \frac{20}{3} + \left|-\frac{4}{3}\right|$

$= 8$ units (1)

1 Mark for being aware of spliting area into 2 parts.

1 mark for $\int_0^4 x^2 - 6x + 8 = \frac{16}{3}$

Question 15

a) (i) $\ddot{x} = 4\sin 2t$

$\dot{x} = \int 4\sin 2t = -\frac{4\cos 2t}{2} + c$

$\dot{x} = -2\cos 2t + c$ (1)

when $t = 0$ $\dot{x} = -1$

$-1 = -2\cos(0) + c$ (1)

$\therefore c = 1$

$\therefore \dot{x} = 1 - 2\cos 2t$

(ii) At rest when $\dot{x} = 0$

$1 - 2\cos 2t = 0$

$\cos 2t = \frac{1}{2}$ — (1)

$2t = \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$

$t = \frac{\pi}{6}, \frac{5\pi}{3}, - -$ (1)

$\therefore$ first at rest when $t = \frac{\pi}{6}$

(iii) $x = \int 1 - 2\cos 2t$

$x = -\sin 2t + t + c_1$

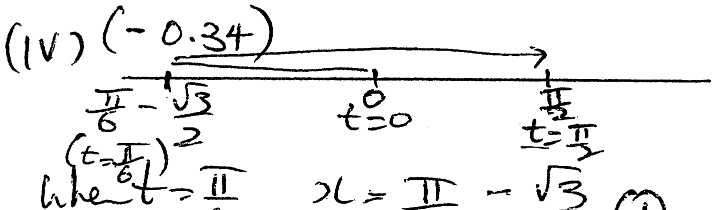
when $t = 0$ $x = 0$

$0 = 0 + 0 + c_1$ (1)

$c_1 = 0$

$\therefore x = t - \sin 2t$

(iv) (-0.34)



when $t = \frac{\pi}{6}$ $x = \frac{\pi}{6} - \frac{\sqrt{3}}{2}$ (1)

when $t = \frac{\pi}{2}$ $x = \frac{\pi}{2} - \sin 0$

$= \frac{\pi}{2}$ $(1.57\ldots)$

$\therefore$ Distance travelled (1)

$2\left|\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right| + \frac{\pi}{2}$ or $[2 \times (0.34\ldots) + 1.57\ldots]$

$= 2\left|-\left(\frac{\sqrt{3}}{2} - \frac{\pi}{6}\right)\right| + \frac{\pi}{2}$

$= \sqrt{3} - \frac{\pi}{3} + \frac{\pi}{2}$

$= \sqrt{3} + \frac{\pi}{6}$ $(2.255\ldots)$ (1)

If answer $\frac{\pi}{2} \rightarrow$ (1)

If $\int_0^{\frac{\pi}{2}} t - \sin 2t \rightarrow$ (1)

1 off each error

b) (i) $e^{1-\ln 2} = e \cdot e^{-\ln 2}$

$= e \cdot e^{\ln\frac{1}{2}}$ (1)

$= e \times \frac{1}{2}$

$= \frac{e}{2}$

(ii) $y' = -4e^{1-4x}$ (1)

when $x = \frac{\ln 2}{4}$ $y' = -4e^{1-\ln 2}$ (1)

$= \frac{-4e}{2} = -2e$

$\therefore$ Eq'n $y - \frac{e}{2} = -2e\left(x - \frac{e}{2}\right)$ (1)

$4ex + 2y - e - 2e^2 = 0$

c) Area $= \int_1^e (\ln 2x - \ln x)\,dx$ (1)

$= \int_1^e \ln 2 + \ln x - \ln x$ (1)

$= \int_1^e \ln 2$

$= \left[x\ln 2\right]_1^e$ (1)

$= e\ln 2 - \ln 2$

$= \ln 2(e - 1)$ (1)

$(1.19\ldots)$

Question 16.

a)(i) $500 = x^2 y \therefore y = \frac{500}{x^2}$ (1)

$A = 4xy + x^2$

$\therefore A = 4x\left(\frac{500}{x^2}\right) + x^2$ (1)

$A = \frac{2000}{x} + x^2$

(ii) Least Area when $A' = 0$ & $A'' > 0$

$A' = -2000x^{-2} + 2x$

$2x - \frac{2000}{x^2} = 0$

$2x^3 = 2000$

$x^3 = 1000$ (1)

$x = 10$

Test $A'' = 4000x^{-3} + 2$

$= \frac{4000}{x^3} + 2$

when $x = 10$ $A'' = 4 + 2$
$= 6 > 0$ (1)

$\therefore$ Least area occurs when $x = 10$

$A = \frac{2000}{10} + 10^2$

$A = 300\ a^2$ (1)

b (i) If A.P. (1)

$2\cos\theta - \sin\theta = 2\sin\theta - 2\cos\theta$

$4\cos\theta = 3\sin\theta$ ---(A)

$\therefore \tan\theta = \frac{4}{3}$ (1)

$\theta = 53°8'$ (1)

(ii) from (A) $4\cos\theta = 3\sin\theta$

$\therefore 2\cos\theta = \frac{3}{2}\sin\theta$ (1)

$\therefore$ sequence becomes

$\sin\theta, \frac{3}{2}\sin\theta, 2\sin\theta$

hence next term is (1)

$\frac{5\sin\theta}{2}$

c)(i) $m_{PA} = \frac{y-6}{x-1}$ (1)

(ii) $m_{PB} = 2m_{PA}$

$\therefore \frac{y-2}{x-3} = \frac{2(y-6)}{x-1}$ (1)

$(y-2)(x-1) = (x-3)(2y-12)$

$xy - y - 2x + 2 = 2xy - 12x - 6y + 36$

$xy - 5y - 10x + 34 = 0$

$y(x-5) = 10x - 34$

$y = \frac{10x-34}{x-5}$ (1)

$= \frac{10(x-5)}{x-5} + \frac{16}{x-5}$

$y = 10 + \frac{16}{x-5}$ (1)

(iii) (1)